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Final Technical Report

for the period

May 1, 1966 through April 30, 1971

on

Real-Time Information-Processing Systems Program

Sponsored by

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Under Contract Number N00014-67-A-0356-0003 Modification AE

and on

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Information-Processing And Control  
SYSTEMS LABORATORY

NORTHWESTERN UNIVERSITY

Evanston, Illinois  
60201

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Technical Report 66-102

Title: ORTHOGONAL PROJECTION AND DISCRETE OPTIMAL LINEAR SMOOTHING

Author: Dr. J. S. Meditch

Date: July 1966

Abstract: The smoothing filter and smoothing error covariance matrix equations are developed for discrete linear systems using the method of orthogonal projection. Two equivalent formulations are presented and found to agree with those of previous authors who had used other methods. The present results in conjunction with the earlier work of Kalman on prediction and filtering give a complete treatment of the discrete linear estimation problem from the viewpoint of orthogonal projection.

Support: The research reported in this document was supported wholly by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03. (Identification Number NR373-502/3-14-66 Electronics Branch).

Remarks: This report was published in SIAM Journal on Control, Vol. 5, No. 1, February 1967, pp. 74-89.

This final technical report on the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) at Northwestern University, Evanston, Illinois under Office of Naval Research Contract Number N00014-67-A-0356-0003 covers the period from May 1, 1966 through April 30, 1971. As specified in the contract, this report is being submitted in lieu of the regular technical report.

The initial contract number was Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch). The period of this contract was from May 1, 1966 through April 30, 1967. The termination date was extended to July 31, 1967 under Office of Naval Research Contract Number N00014-67-A-0356-0003, Modification AB, and to November 30, 1967 under Modification AC.

Sponsorship of the program was then assumed by the Advanced Research Projects Agency, under ARPA Order No. 1129, although the office of Naval Research continued to monitor the program under Contract N00014-67-A-0356.

The termination date was extended to November 30, 1969 at that time, under Modification AE of the contract. It was later extended to March 31, 1970 by Modification H, and then to April 30, 1971 by Modification 10.

The purpose of the contract was to support research in modern electronics in Northwestern University's Information-Processing And Control (IPAC) Systems Laboratory. The program of this Laboratory initially encompassed the following areas of research: (a) Systems theory and techniques, including pulse modulation and switching, steady-state and dynamic optimization, suboptimal control, learning control, optimum nonlinear estimation and control, and computational methods for optimal control; (b) Information-Processing and computer sciences, including pattern recognition, sequential machines and automata, signal transmission and information processing in physiological systems, threshold logic, graph theory and linear group codes, analysis and optimum design of communication networks, and organizational problems of very high speed computers; (c) Physical electronics, which includes solid-state devices and liquid-state phenomena and devices and gaseous electronics; and (d) Electromagnetic fields, covering coherence theory and mode coupling, boundary value problems, and reflection and scattering and antenna systems.

The program was an interdisciplinary effort conducted by the technical staff of the IPAC Systems Laboratory during the first fifteen months of the contract. Nineteen faculty members belonged to the technical staff. Ten faculty members and eleven graduate students received support for research during the first fifteen months of the contract. Five doctoral theses resulted from the graduate-student research.

The funds in the contract were exhausted after fifteen months in July, 1967. The contract was extended on a no-cost basis until it was refunded in December, 1967.

At that time it was decided to strengthen the research effort in the IPAC Systems Laboratory by concentrating on the fundamental principles of physics pertinent to the system of interest and then designing new components and/or systems to achieve the desired physical results. The initiation of this plan was implemented with the beginning of the program of research in real-time information-processing systems, under Modification, AE of the contract.

RESEARCH IN PROGRESS ON APRIL 30, 1971

ELECTRON-BEAM READOUT OF ACOUSTIC SURFACE WAVES Prof. M. Epstein, N. Zuefle

The objective of this research program was the study of new methods for detection of acoustic surface waves. In order to provide a variable and controllable delay of signals, the technique of electron-beam sensing of surface waves was devised. It is based on the modulation of secondary electron emission from strained surfaces of piezoelectric crystals when the latter serve as substrates for the propagating acoustic surface wave.

The apparatus commonly used in electronic image storage was designed and constructed. Experiments with a surface-wave device designed for a 6MHz pulsed carrier frequency indicated that the secondary electron current is modulated by the signal envelope of the elastic surface wave. Although such detection of the video signal is desirable, the apparent rectification of the rf pulse requires explanation, and a satisfactory explanation has not yet been found. A minor modification of the apparatus was sufficient to show that the rf modulated carrier can also be obtained. However, a major change in the experimental system, primarily the design of an electron-beam source capable of producing a spot of small size, is required to continue this investigation. Such a system is now being designed and constructed.

If this effort is successful, a device using electron-beam sensing of elastic surface waves can be built for application in signal processing and storage (e.g., in matched filters and random access circulating memories.)

Initial results of this work were reported at the European Microwave Conference in London, England in September 1969 and were published in the Proceedings of IEEE Letters in February 1970.

WIDEBAND TRANSDUCTION TECHNIQUES FOR ACOUSTIC SURFACE-WAVE DEVICES Prof. M. Epstein; M. E. Motamedi

The purpose of this research program was the study of electro-acoustic transduction on piezoelectric crystals leading to elastic surface-wave devices which are capable of processing wideband signals.

Elastic surface-wave transducers were fabricated utilising photolithographic techniques. (As part of this research program and partly due to the support received under this contract, a complete facility for photoresist fabrication was established.) Structures consisting of log-periodic arrays of interdigital electrodes were fabricated directly on substrates of  $\text{LiNbO}_3$  and glass. The interdigital transducers fabricated on glass substrates were employed in the investigation of mechanically variable surface-wave delay lines.

Fourier analysis of an approximate electric field distribution at the location of the interdigital transducer was found to provide a useful method for the synthesis of elastic surface-wave devices with specified filter characteristics. The analytical model showed remarkably good agreement with experimental results. The performance of the log-periodic interdigital transducer in terms of frequency response and impedance was shown to be considerably better than that of conventional resonant (uniform) interdigital transducers. Wideband and resonant interdigital transducers fabricated on glass substrates were used to generate and detect elastic surface waves propagating on  $\text{LiNbO}_3$  crystals. It was shown that such a technique can lead to the design of mechanically variable delay lines with additional insertion loss not exceeding 14 db per transducer.

In the course of this study, a method of insertion loss measurement of transducers was devised which eliminates the effects of the measuring apparatus. It is based on the fact that the measured signal at the output transducer of an acoustic surface-wave delay line can be related to the radiation resistance of the input transducer.

The results of this work were published in the Proceedings of IEEE Letters in November 1970.

THIN-FILM SURFACE-WAVE TRANSDUCERS Prof. M. Epstein; F. C. Luo

The objective of this research program was the investigation of the possible application of the strain-sensitivity of thin-film field-effect transistors to the transduction of elastic surface waves. The nonlinear characteristics of such a device and their possible utilization in signal detection were of special interest.

Thin-film field-effect transistors were fabricated of cadmium sulphide and cadmium selenide on glass substrates. The acoustic surface waves were generated by using the techniques of mechanically variable delay lines (as described in the section on Wideband Transduction Techniques for Acoustic Surface-Wave Devices).

Although thin-film transistors with high transconductance were obtained ( $gm \approx 10,000$  micromhos), it appears that surface-wave transduction and field-effect amplification are incompatible; i.e., the desired characteristics of the material (high piezoelectric coupling and field effect) cannot be optimized simultaneously.

THIN-FILM SURFACE-WAVE ACOUSTIC AMPLIFIER Prof. M. Epstein; F. C. Luo

The purpose of this research program was the investigation of surface-wave amplification by means of interaction of elastic waves in a piezoelectric substrate with drift electrons in a thin-film semiconductor. The major features of such a device reside in the possibility of obtaining very thin films of semiconductor with high mobility of carriers.

During the present research program thin films of indium antimonide were prepared by vacuum deposition and recrystallization in an inert atmosphere of high-purity argon. Thin films of InSb (500 - 1000 Å) on glass were obtained with Hall mobilities ranging between 1,200 and 2,500 cm<sup>2</sup>/v-sec. This work is currently being continued with the objective of depositing such films on piezoelectric substrates of PZT ceramics.

IMPEDANCE CHARACTERIZATION OF TRANSDUCERS Prof. M. Epstein; A. Nalamwar

The objective of this study was to develop improved methods of characterization of electroacoustic transducers. An analytical technique was used to determine the impedance of an elastic surface-wave transducer from insertion loss measurements. It is based on the Real Part Sufficiency and the properties of Hilbert transforms.

Comparison of analytical results based on digital computation with experimental data showed that the susceptance or reactance of a transducer can be evaluated with far greater accuracy than is possible in most experimental techniques. Since the static capacitance of a transducer can be measured quite accurately at low frequencies, it appears that the present method can become significant in obtaining a more accurate equivalent circuit for the transducer.

INTERACTIVE COMPUTER GRAPHICS TERMINAL Prof. B. W. Jordan

During this contract, research and development on a new concept for an interactive computer graphics display were pursued. This concept has been labelled the "cell display" concept and represents a generalization of the familiar raster display in which the display is divided into character cells, each of which is a matrix of dots. The organization of the screen is then in terms of character rows instead of rasters. This concept allows the use of standard TV monitors, with their inherent advantage over XY displays while retaining the ability to describe and generate a picture using "primitives", a technique previously available only to XY displays.

The hardware blocks required to generate the display consist of a memory system for the display file, a character generator, a display output buffer and a TV monitor. The main area of concentration has been the character generator. Two basically different design approaches were investigated. The first, which was reported on in detail in the Quarterly Technical Report dated March, 1970, was a combinatorial logic approach in which a character is generated from primitives through a combinatorial logic network. The final results of this design are available in reference 1. The second approach made use of micro-programming concepts. For this design, the primitives passed only through that portion of the combinatorial network required by the instructions from the display file. This routing is determined by the microprogram executed. The details of this design were reported in the Quarterly Technical Report dated December, 1970. The final results of this design approach are available in reference 2.

In addition to the hardware design, the translation process from the more natural XY picture descriptions to the cell format and the selection of primitives have been investigated. These problems are somewhat intertwined since the translation algorithms and the resulting picture quality depend upon the available primitives. A description of the problems involved and the method of attack on those problems was presented in the Quarterly Technical Report dated September, 1970. A simulation of the system has been programmed on a general purpose computer with a storage tube display, and the various algorithms and primitives are being investigated using this simulation. To date, a generalization of an algorithm given by Metzger, reference 3, seems to be producing the highest quality of figures. A set of primitives which give results identical to those obtained when complete freedom is available for the position of each dot has been devised.

References:

1. McCracken, W.T., "Design of the Character Generator for a Computer Graphics Terminal," M.S. Thesis, Dept. of Electrical Engineering, Northwestern University, January, 1971.
2. Thieler, S.M., "Design Parameters Affecting a Microprogram Controlled Character Generator," M.S. Thesis, Dept. of Electrical Engineering, Northwestern University, May, 1971.
3. Metzger, R.A., "Computer Generated Graphic Segments in a Raster Display," Proceeding of the Spring Joint Computer Conference, 1969.

PATENT DISCLOSURE

A new concept in the design of surface-wave transducers has been conceived. The proposed structures appear to offer new methods in the design of electrically matched transducers and should result in simplified fabrication techniques. A patent disclosure describing the invention was filed with the legal office of the University.

TECHNICAL REPORTS AND PAPERS PUBLISHED

Attached is a list of individual technical reports and papers based on research supported under this contract. The degree of support from this contract is indicated for each report or paper.

Office of Naval Research  
Contract Number N00014-67-A-0356-0003  
(Original Contract Number was N00014-66-C0020-A03)

Technical Reports and Papers Published

Technical Report 66-101

Title: DETERMINATION OF EXPECTED LONGEST CARRY PROPAGATION LENGTHS FOR BINARY ADDITION OF SIGNED NUMBERS

Authors: C. C. Yang and Dr. S. S. Yau

Date: June 1966

Abstract: The expected longest carry delay of asynchronous self-timing additions and the expected maximum number of processing cycles of synchronous iterative additions for signed binary numbers are exactly determined. The former depends on the longest carry propagation lengths due to either zero or nonzero carries, whereas the latter depends only on the longest nonzero carry propagation lengths. The signed numbers are represented by 2's complement notation. It is found that the enormous possible combinations for binary additions of signed numbers and their longest carry delays can be expressed by some recursive formulas if state representations of allowable addition processes (without overflow) are employed. By this method, efficient computer programmings are written to determine exactly the expected longest carry delays for asynchronous additions and the expected maximum numbers of processing cycles for synchronous iterative additions. The results for  $n = 2$  through  $n = 48$ , where  $n$  is the number of bits of summands, are given. The dependence relations of these delays on  $n$  are also considered.

Support: The research reported in this document was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

Technical Report 66-102

Title: ORTHOGONAL PROJECTION AND DISCRETE OPTIMAL LINEAR SMOOTHING

Author: Dr. J. S. Meditch

Date: July 1966

**Abstract:** The smoothing filter and smoothing error covariance matrix equations are developed for discrete linear systems using the method of orthogonal projection. Two equivalent formulations are presented and found to agree with those of previous authors who had used other methods. The present results in conjunction with the earlier work of Kalman on prediction and filtering give a complete treatment of the discrete linear estimation problem from the viewpoint of orthogonal projection.

**Support:** The research reported in this document was supported wholly by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03. (Identification Number NR373-502/3-14-66 Electronics Branch).

**Remarks:** This report was published in SIAM Journal on Control, Vol. 5, No. 1, February 1967, pp. 74-89.

Technical Report 66-103

Title: ASSOCIATIVE MEMORY SYSTEMS AND THEIR APPLICATIONS TO PICTURE AND ARITHMETIC PROCESSINGS

Author: C. C. Yang

Date: August 1966

**Abstract:** Two different associative memory systems implemented by cryotrons and cutpoint cells are presented. The cutpoint cellular associative memory designed on the basis of stage delay analysis does not use any ladder structure, but employs a high-speed controlled shift register for the sequential tagging of all matched memory words. Both systems are so mechanized that their structures are simple and especially compatible with the batch fabrications of integrated circuits. Moreover, both systems can perform ordered information retrieval following Lewis's sorting scheme, template-matching pattern recognition based on the union-intersection concept and nonbulk-two summand additions of signed binary numbers in the signed-2's-complement number system. The basic operations of each system include comparison, reading, writing, sensing and output-summing processes being all performed on the basis of parallel-by-bit. During simultaneous comparison, a bit-position corresponding to all bit-memories of equal significance can be masked out. In pattern recognition, each pattern class is allowed to have deviations in size, style, orientation, etc. within certain limits. Since each prestored template for a pattern class can accommodate to a number of deviated input patterns belonging to the same class, the storage capacity can be greatly saved. Because most input patterns are recognized within first processing cycle and no patterns require more than two comparison processes for their recognition, the pattern recognition scheme is simple. The speed is very high because of this simple scheme as well as the parallel processing capability of an associative memory. The processing system for arithmetic operations consists of an associative memory, right shift registers and Mealy type sequential machines. This system may have the advantage of reducing addition time by paying a limited price in the additional hardware relative to the existing software approaches for arithmetic algorithms.

**Support:** The research reported in this document was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

Technical Report 66-104

Title: SOME GENERALIZATIONS OF THE DISCRETE OPTIMAL CONTROL PROBLEM

Author: Dr. S. G. Bankoff

Date: October 1966

Abstract: Some modifications of the Kuhn-Tucker constraint qualifications are given for non-linear programming problems where non-linear equality constraints are present. The treatment by Rosen of optimal discrete control problems is then generalized to include cyclic topologies. In addition, an approach to the discrete analog of the minimum-time problems is outlined.

Support: The research reported in this document was supported wholly by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

Remarks: Parts of this paper were published in Int. J. Control, Vol. 7, No. 5, 1968, pp. 447-449.

Technical Report 66-105

Title: GENERATION OF AN OUTPUT REGULAR EXPRESSION OF A SEQUENTIAL MACHINE WITH A SPECIFIED INPUT REGULAR EXPRESSION

Author: Dr. S. S. Yau

Date: October 1966

Abstract: It has been shown that the set of output sequences of a sequential machine due to a regular set of input sequences is regular. However, there is no method of generating the regular expression for the output regular set. The purpose of this paper is to present such a technique which is based on constructing the output graph from the state graph of the sequential machine and the input regular expression. Once the output graph is obtained, the regular expression can readily be obtained by using the signal flow graph techniques proposed by Brzozowski and McCluskey. The technique presented in this paper can be applied to both Moore and Mealy machines.

Support: The research reported in this document was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

Remarks: This report was published in the Proceedings of the Allerton Conference on Circuit and Systems Theory, 1966.

## Technical Report 66-106

Title: A CRYOGENIC ASSOCIATIVE MEMORY SYSTEM FOR INFORMATION RETRIEVAL

Authors: Dr. S. S. Yau and C. C. Yang

Date: November 1966

**Abstract:** The cryogenic associative memory system presented here can perform both ordered and non-ordered information retrieval, which includes the comparison, writing, reading and sensing processes. The reading process can be either non-destructive or destructive. In ordered information retrieval, Lewin's sorting scheme, whose number of interrogation cycles is minimum among all the sorting techniques so far available, is employed. The system may contain and distinguish the contents "0", "1" and the empty "e" in each memory cell. The selection and control circuit for the whole cryogenic associative memory system is simple because the operations of all the processes in the cryogenic circuit are automatic. The cryotrons used in every memory cell and in the cryogenic selection and control circuits for each memory word are fewer than the existing systems for performing the equivalent functions. The operating speed of the proposed cryogenic associative memory system is increased by reducing the circuit inductances and minimizing the number of stage delays.

**Support:** The research reported in this document was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C00020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch.)

**Remarks:** This report was published in the Proceedings of the National Electronics Conference, Vol XXII, pp. 764-769 (1966).

## Technical Report 66-107

Title: ON A SPECIAL CLASS OF CONSTRAINED GENERALIZED MEDIAN PROBLEMS

Authors: Dr. A. Charnes, Dr. M. J. L. Kirby (University of Chicago) and W. M. Raike

Date: November 1966

**Abstract:** By means of elementary properties of the absolute value functions, important properties of a special class of "constrained generalized median" problems (and eventually, the most general class, *vide Charnes, Cooper, Thompson*) such as existence of solutions, gradient and incremented formulae, linear programming and probabilistic interpretations are obtained for all classes of joint distribution functions for which the problems make sense. Results of A. C. Williams and R. Wets obtained by involved arguments and sophisticated constructs appear, when corrected, as special instances of some of the above results but devoid of the interrelations and interpretations herein adduced.

**Support:** The research described in this report was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373 502/3-14-66 Electronics Branch) and in part by the U. S. Army under U. S. Army Research Office - Durham Contract Number DA-31-124-ARO-D-322.

Technical Report 66-108

**Title:** OPTIMAL FIXED-POINT CONTINUOUS LINEAR SMOOTHING

**Author:** Dr. J. S. Meditch

**Date:** December 1966

**Abstract:** The filter and error covariance equations for optimal fixed-point smoothing for continuous linear systems are developed. The development is carried out by considering the limiting case of the results for the same problem for discrete linear systems. The procedure is of use in estimation problems where a smoothed estimate of a continuous linear system's state is desired at some specified critical time during the system's operation. Four examples are presented to illustrate the results.

**Support:** This work was supported wholly by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014 66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

**Remarks:** This report was published in the Proceedings of the 1967 Joint Automatic Control Conference, University of Pennsylvania, Philadelphia, Pennsylvania, June 28-30, 1967.

Technical Report 67-101

**Title:** ON BALANCED SETS, CORES, AND LINEAR PROGRAMMING

**Authors:** Dr. A. Charnes and Dr. K. Kortanek (Cornell University)

**Date:** January 1967

**Abstract:** L. S. Shapley has found a necessary and sufficient condition for the non-emptiness of the core of a characteristic function n-person game stating that the core is non-empty if and only if a certain system of linear inequalities on minimal balanced collection of finite sets is consistent. Using some well known constructs of linear programming, we associate to any n-person game two dual

Linear programming problems in which the constraint set of the primal includes the core of the game, and characterize the non-emptiness of the core in terms of properties of dual optimal solutions of these problems. We then prove the Shapley conjecture on sharpness of the set of proper minimal balanced inequalities with respect to core feasibility of proper n-person games. Using the Farkas-Minkowski Theorem, we obtain a characterization of redundant inequalities with respect to core feasibility and express the rate of growth of the game as a sequence of lower bounds for successive game values corresponding to increasing subsets of the collection of N players, which vitiates the possibility of constraint redundancy. If all game values are non-negative, the characteristic growth rate induces a partial ordering on game values corresponding to subsets of N.

**Support:** This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch) at Northwestern University and in part by the Pilot Program in Environmental Systems Analysis under National Institute of Health Grant 1 PIO ES 00098-01 at Cornell University.

**Remarks:** This report was published in Centre D'Etudes De Recherche Operationnelle (Institut De Statistique De L'Universite Libre De Bruxelles), Vol. 9, No. 1, 1967.

**Technical Report 67-102**

**Title:** REALIZATION OF A CLASS OF SWITCHING FUNCTIONS BY THRESHOLD LOGIC NETWORKS

**Authors:** Dr. S. S. Ya. and D. L. Ostapko

**Date:** January 1967

**Abstract:** In this paper, a class of switching functions, called threshold-product functions, whose definition is analogous to that of threshold functions (which will be called threshold-sum functions), is studied in detail. It is shown that both threshold functions and parity functions are special cases of threshold-product functions. A simple and economical threshold-logic realization method is established for threshold-product functions. This economical realization method is based on a constrained solution of threshold-product functions. A systematic technique for finding a constrained solution for a threshold-product function is obtained, and this technique can be employed for testing whether a switching function is a threshold-product function as well. When the number of variables in a switching function is not large, say no more than 6, a simpler method for the above purposes is found. Furthermore, a threshold-logic realization method which yields a minimal realization for certain threshold-product functions is obtained.

Support: This work was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

Technical Report 67-103

Title: ANALYSIS AND DESIGN OF COMMUNICATIONS NETWORKS WITH MEMORY

Author: Dr. S. L. Hakimi

Date: January 1967

Abstract: A mathematical formulation of the communication networks with memory is presented assuming that the sources of traffic are deterministic but not necessarily time invariant. The formulations lead to a linear programming problem. Some generalizations and justification of the choice of the model are discussed. The same basic formulation can be used as a tool for analysis as well as least-cost design or improvement of an existing network. Design of the memory systems and its relation with messages with priorities is considered. Similar concepts are used to arrive at an approximate linear programming formulation of "street traffic".

Support: This work was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

Remarks: Part of this report was published in the Proceedings of the First Annual Princeton Conference on Information Sciences and Systems, March 1967.

Technical Report 67-104

Title: ON SHIFT-REGISTER REALIZATIONS OF SEQUENTIAL MACHINES

Author: C. C. Su

Date: January 1967

Abstract: The problem of determining secondary state assignments for sequential machines such that the binary memory elements are connected in the form of shift registers is studied. Algorithm for finding such state assignments is developed. One or more code words may be assigned to a state of the sequential machines. The only restriction is that the realizations be unitary. A single shift-register realization of a sequential machine is unitary if and if all code words assigned

to a state have the same first digit. In a multiple shift-register realization of a sequential machine, corresponding to each shift register, there exists a set system on the state set of the sequential machine. A multiple shift-register realization is unitary if each of the set system is a partition and in other words assigned to a block of the partitions have the same first digit. With our technique the unitary realizations consisting of the least number of shift registers can be found for any finite, deterministic, synchronous and reduced (minimal state) sequential machine, each of whose states has a non-empty predecessor set. The algorithm is suitable for programming on digital computers.

**Support.** This work was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

Technical Report 67-105

**Title:** ON OPTIMAL LINEAR SMOOTHING THEORY

**Author:** Dr. J. S. Meditch

**Date:** March 1967

**Abstract:** The algorithm for generating the smoothed estimate  $\hat{x}(t|t+T)$  of the state  $x(t)$  of a continuous linear system, where  $t$  is continuous time,  $T$  is a positive real constant, and  $t+T$  is the time of the most recent measurement, is developed. A linear matrix differential equation whose solution is the covariance matrix of the smoothing error  $\tilde{x}(t|t+T) - \hat{x}(t|t+T)$  is then derived. Computational aspects involved in mechanizing the algorithm are discussed in terms of the algorithm's dependence on the solution of the prediction, filtering, and fixed-point smoothing problems. The results are then discussed in terms of the classical Wiener smoothing problem.

**Support:** This work was supported wholly by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch).

**Remarks:** This report was published in the Journal of Information and Control, January 1968.

Technical Report 67-106

**Title:** FEASIBILITY OF USING LINEAR PATTERN CLASSIFIERS FOR PROBABILISTIC PATTERN CLASSES

**Authors:** Dr. S. S. Yau and P. C. Chuang

**Date:** March 1967

**Abstract:** A linear pattern classifier is a pattern classifier which employs linear discriminant functions. Because of the simplicity and low cost in the implementation of a linear pattern classifier, one always prefers to use such a pattern classifier whenever it is feasible. Since most pattern classes in reality are of probabilistic nature and not linearly separable, one accepts a small percentage of misclassification of patterns in order to take the great advantage of using a linear pattern classifier. The purpose of this correspondence is to find a condition for the existence of a linear pattern classifier to classify patterns of two classes within specified percentage of misclassification.

**Support:** This work was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Defense Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number NR 373-502/3-14-66 Electronics Branch)

**Remarks:** This report was published in the IEEE Proceedings, Vol. 54 No. 12 December, 1966, pp. 1957-1959.

Technical Report 67-107

**Title:** A NONBULK ADDITION TECHNIQUE FOR ASSOCIATIVE PROCESSORS

**Authors:** Dr. S. S. Yau and C. C. Yang

**Date:** April 1967

**Abstract:** So far, almost all the work in applying associative processors to perform arithmetic operations belongs to the software approach with the bottleneck characteristic. In this note, a nonbulk addition technique, which utilizes the sensing process of an associative memory, is presented. The hardware required to implement this technique is very simple. In addition to an associative memory, it requires two shift registers, each with  $n+2$  flip-flops, and a sequential machine with two input terminals, two output terminals and six states. The addition speed can be increased by using a method similar to that in the conditional sum adder proposed by Sklansky, but the rate of increase of hardware in our system is much smaller.

Support: This work was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number AF 373-502/3-14-66 Electronics Branch).

Remarks: This report was published in the IEEE Transactions on Electronic Computers, December 1966.

### Technical Report 67-108

Title: PATTERN RECOGNITION BY USING AN ASSOCIATIVE MEMORY

Authors: Dr. S. S. Yau and C. C. Yang

Date: April 1967

Abstract: The purpose of this paper is to present a simple template matching pattern-recognition technique by using any general-purpose associative memory. The input patterns for recognition may have wide variations, provided that the distinct features of individual pattern classes can be extracted. Each pattern class is allowed to have deviations in size, style, orientation, etc. within certain limits. This pattern-recognition technique is extremely efficient in hand-written character recognition, which is used for illustration in this paper. Because each input pattern is processed with all the pattern classes simultaneously, the speed of this pattern-recognition technique is very high. It is found that most input patterns are recognized within first comparison process and no input patterns require more than two comparison processes for their recognition.

Support: This work was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03 (Identification Number AF 373-502/3-14-66 Electronics Branch).

Remarks: This report was published in the IEEE Transactions on Electronic Computers, December 1966.

## Technical Report 67-109

**Title:** PLANE WAVE DIFFRACTION FROM A SMALL HALF-SPHERE MOUNTED ON A LARGE CONDUCTING BODY

**Author:** Dr. M. A. Plonus

**Date:** May 1967

**Abstract:** Using image theory and the scattering solution for a sphere, the plane wave scattering by a small, perfectly conducting half-sphere on a conducting plane is obtained first. The scattered field can then be identified as that radiated from a combination of electric and magnetic dipoles on the plane. Equating these dipole fields to the fields of the corresponding radiation dipoles, we can solve for the induced scattering dipole moments. These can now be used to obtain the scattering of a small half-sphere mounted, for example, on large cones, spheres, cylinders, etc., provided the simpler problem of radiation dipoles on the large bodies has been solved first.

**Support:** This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003, Modification AB and in part by the National Science Foundation under Grant GK-1350.

**Remarks:** This report was published in the Proceedings of the IEEE, Vol. 55, No. 2, February 1967, pp. 214-216.

## Technical Report 67-110

**Title:** GRAPH THEORETIC ERROR-CORRECTING CODES

**Authors:** Dr. S. L. Hakimi and J. G. Bredeson

**Date:** June 1967

**Abstract:** This paper presents a study of graph theoretic block codes, their efficiency, their error correcting capabilities, and their limitations. Augmentation of graph theoretic codes and their generation are discussed. It is shown that such augmentation techniques often can increase the level of efficiency of these codes to that of the best available codes. Furthermore, the augmented graph theoretic codes are shown to be easily decodable. The same ideas are applied to the directed graphs resulting in a class of easily decodable ternary codes.

**Support:** This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003, Modification AB and in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65.

**Remarks:** This report was published in the IEEE Transactions on Information Theory, Vol. IT-4, No. 4, July, 1968.

## Technical Report 67-111

Title: THE WIENER-HOPF SOLUTION OF THE OPTIMAL FIXED POINT SMOOTHING PROBLEM

Author: Dr. J. S. Meditch

Date: June 1967

Abstract: The algorithm for optimal fixed point data smoothing for continuous linear systems is developed by solving the appropriate Wiener-Hopf matrix integral equation in the time domain. Two formulations are given for generating the optimal smoothing filter gain matrix, and three matrix ordinary linear differential equations whose solutions yield the smoothing error covariance matrix are derived. The results substantiate and extend those obtained by the author earlier via another method.

Support: This work was supported wholly by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 Modification AB.

Remarks: This report was published in the International Journal of Control, January 1968.

## Technical Report 67-112

Title: OPTIMAL CONTROL OF DISTRIBUTED PARAMETER CHEMICAL REACTORS

Author: Dr. K. S. Chang

Date: July 1967

Abstract: The synthesis of optimal control of distributed parameter processes governed by a system of simultaneous partial differential equations of the first-order in two independent and  $n$  dependent variables is considered. The theory on necessary conditions for optimal control, the computational algorithm based on the first-order variations, and the method of characteristics are applied to the optimal control of a plug-flow tubular reactor with non-linear chemical kinetics. Optimal response to oscillatory feed disturbances is also investigated. In some cases it may be desirable to operate under intentionally oscillatory input conditions. Extensive numerical computations are presented.

Support: This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 Modification AB and in part by the National Science Foundation under Grant Number GP-2175.

Technical Report 67-113

Title: A HAND-PRINTED CHARACTER RECOGNITION EXPERIMENT

Authors: W. J. Steingrundt and Dr. S. S. Yau

Date: August 1967

**Abstract:** The experimental results for hand-printed character recognition based on line-pattern recognition as a second order Markov process are presented. After collecting 2,000 sample patterns for 10 hand-printed characters, this method shows 89.1% correct classification for patterns not in the training set. Some improvements for this approach are suggested.

**Support:** This work was supported in part by the U. S. Air Force Office of Scientific Research, Grant AF-AFOSR-98-65, and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 Modification AB.

**Remarks:** This report was published in the Proc. First Annual Princeton Conference on Information Sciences and Systems, March 1967.

Technical Report 67-114

Title: SCATTERING FROM LOW-DENSITY DIELECTRIC BODIES

Author: Dr. M. A. Plonus

Date: September 1967

**Abstract:** It is shown that the coherent scattering integral provides a convenient and mathematically tractable method of solving scattering problems of large, low-density, dielectric bodies. In this method the solid dielectric body is replaced by an assembly of particles whose boundary coincides with the outside surface of the dielectric body. Considering the coherent scattering integral, and realizing the coherent scattering comes primarily from sudden particle density changes such as the boundaries of a particle system, it is shown that for low-density dielectric bodies of arbitrary shape the coherent scattering integral yields the physical optics cross section of the dielectric body. Back-scattering echo areas for a cylinder, sphere, lens, and various truncated cones are derived and discussed.

**Support:** This work was supported in part by the National Science Foundation under Grant GK-1350, in part by the Air Force Office of Scientific Research under Contract AF 49(638)-1377, and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 Modification AB.

**Remarks:** This report was published in IEEE Trans. on Antennas and Propagation, vol. AP-15, number 3, May 1967, pp. 437-441.

## Technical Report 67-115

Title: SPACE FILLING CURVES AND MATHEMATICAL PROGRAMMING

Author: Dr. A. R. Butz

Date: November 1967

**Abstract:** The problem of finding  $\bar{x}$  in a  $n$  dimensional Euclidean space such that  $f_i(\bar{x}) \leq 0$ ,  $i = 1, 2, \dots, N$ , is considered. The only assumption on the  $f_i$  is that a solution exists in the quantized unit hypercube. Implicitly exhaustive solution procedures, which obtain solutions by implicitly considering every point in the quantized space without making computations at each point, are studied. The implicitly exhaustive feature is made possible by adapting "space filling curves" to discrete spaces of general dimensionality. Several space filling curves are surveyed, and Peano's continuous mapping from the unit interval onto the unit square is used as a basis for defining a mapping from the unit quantized interval onto the unit quantized hypercube, and vice versa. Ternary arithmetic is the basis for the required functional relationships in the discrete mapping. The discrete mapping has attributes of global continuity, and specific numerical bounds are derived in this respect. It is shown that these bounds are of optimal order dependence on the relevant variables. It is shown how to use knowledge of first and second order local properties of the  $f_j$  to obtain solutions on a digital computer using space filling curves as a basis for the concept of implicitly exhaustive search. No knowledge of global properties is assumed.

**Support:** This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research, Contract Number N00014-67-A-0356-0003 Modification AB, and in part by the National Science Foundation under Grant Number GK-1540.

**Remarks:** This report was published in the Journal of Information and Control, Vol. 12, No. 4, April 1968.

## Technical Report 67-116

Title: DETERMINATION OF LINEAR DISCRIMINANT FUNCTIONS

Author: Dr. A. R. Butz

Date: November 1967

**Abstract:** The problem of determining a linear discriminant function for approximately separating two sets of points in a space  $E^m$  is treated. The two sets of points are assumed not linearly separable; hence only an approximation is possible.

Support: This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 Modification AB and in part by the National Science Foundation under Grant Number GK-1540.

Technical Paper: DECODING OF GRAPH THEORETIC CODES

Authors: Dr. S. L. Hakimi and J. G. Bredeson

Abstract: The purpose of this correspondence is to show that graph theoretic codes are threshold, or more specifically majority decodable. In a forthcoming paper, we intend to present a rather complete study of graph theoretic codes, their efficiency, their error correcting capability, their limits and their generalizations.

Support: This work was supported in part by the Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract N00014-66-C0020-A03.

Remarks: This paper was published in the IEEE Transactions on Information Theory, Vol. IT-13, No. 2, April 1967, pp. 348-9.

Technical Paper: STOCHASTIC APPROXIMATION NONPARAMETRIC TRAINING PROCEDURES FOR MULTI-CATEGORY PATTERN CLASSIFIERS.

Authors: Dr. J. M. Schumpert and Dr. S. S. Yau

Abstract: In this paper, two nonparametric training procedures for multi-category pattern classifiers are proposed. These two training procedures are based on the Robbins-Monro stochastic approximation procedure. The computation required is very simple. The iterative property of these two procedures enables the pattern classifier to be adaptive when the pattern classes are time-variant. Several examples are simulated and their results converge to local optima.

Support: This work was supported in part by the U. S. Air Force Office of Scientific Research under Grant Number AF-AFOSR-98-65 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract N00014-66-C0020-A03.

Remarks: This paper was presented at the Fifth Annual Allerton Conference on Circuit and System Theory, October 4, 1967, and appears in its proceedings.

Technical Paper: LEARNING BANG-BANG REGULATORS

Author: Dr. A. R. Butz

**Abstract:** A continuous time algorithm for learning a stable bang-bang regulator is developed. It is assumed that a Lyapunov function  $V(x)$  is known such that  $V$  is negative definite for some bang-bang control. The plant may be unstable since a simple condition (that a constant be large enough) of the learning algorithm guarantees stability of the closed loop system during the learning phase. Derivative computation is required, but a significant time delay and an error are permissible in such computation.

**Support:** This work was supported in part by the National Aeronautics and Space Administration under Grant NGR-24-005-065 to the University of Michigan and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract N00014-66-C-A03.

**Remarks:** This paper was published in the IEEE Transactions on Automatic Control, Vol. AC-13, No. 1, February 1968, pp. 96-99.

Technical Paper: UNITARY SHIFT-REGISTER REALIZATION OF SEQUENTIAL MACHINES

Authors: C. C. Su and Dr. S. S. Yau

**Abstract:** The problem of determining secondary state assignments for sequential machines such that the binary memory elements are connected in the form of shift registers is studied. Algorithm for finding such state assignments is developed. One or more code words may be assigned to a state of the sequential machine. The only restriction is that the realizations be unitary. A single shift-register realization of a sequential machine is unitary if and only if all code words assigned to a state have the same first digit. In a multiple shift-register realization of a sequential machine, corresponding to each shift register, there exists a set system on the state set of the sequential machine. A multiple shift-register realization is unitary if each of the set system is a partition and if code words assigned to a block of the partitions have the same first digit. With our technique, the unitary realizations consisting of the least number of shift registers can be found for any finite, deterministic, synchronous and reduced (minimal-state) sequential machine, each of whose states has a non-empty precessor set. The algorithm is suitable for programming on digital computers.

**Support:** This work was supported in part by the U. S. Air Force Office of Scientific Research under Grants AF-AFOSR-98-65 and AF-AFOSR-1292-67 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03.

**Remarks:** This paper was published in the IEEE Transactions on Computers, Vol C-17, No. 4, April 1968, pp. 312-324.

Technical Paper: SYSTEM EQUIVALENT OPTIMAL CONTROL PROBLEMS

Authors: W. J. Rugh and Dr. G. J. Murphy

**Abstract:** Some new equivalence properties of optimal control problems are given in this paper. The phase-variable canonical form is used to define the various optimization problems, involving linear plants with quadratic performance indices, which yield identical closed-loop systems. These results are then used to investigate the structure of the finite-terminal-time problem in relation to the infinite-terminal-time problem.

**Support:** This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 and in part by the Advanced Research Projects Agency of the Department of Defense under the same contract, monitored by the Office of Naval Research.

**Remarks:** This paper was presented at the Sixth Annual Allerton Conference on Circuit and System Theory, October, 1968, and was published in the Proceedings of that Conference.

Technical Paper: OSCILLATORY OPERATION OF JACKETED TUBULAR REACTORS.

Authors: Dr. K. S. Chang and Dr. S. C. Bankoff

**Abstract:** Optimal control is studied of the cooling jacket water temperature of an empty tubular reactor in which consecutive (Bilous-Amundson) reactions are carried out, subject to sinusoidal perturbations of the feed concentration. In this particular system, second-order effect on the desired outlet concentration were small, and could be either positive or negative, depending upon the frequency and parametric effects. The improvements seemed to be principally due to the zero-frequency effect, based upon the curvature of the steady-state response surface to inlet concentration, using optimal cooling water temperatures. Dynamic effects decreased the performance. Parametric studies were made of the effects of heat transfer coefficient and heat of reaction.

**Support:** This work was supported in part by the National Science Foundation under Grant GK-1126 and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003.

**Remarks:** This paper was published in the Industry & Engineering Chemistry Fundamentals, Vol. 7, November 1968, pp. 633-639.

Technical Paper: A NEW APPROACH TO THE SOLUTION OF LINEAR OPTIMAL CONTROL PROBLEMS

Authors: W. J. Rugh and Dr. G. J. Murphy

**Abstract:** A simplified approach to the solution of linear optimal control problems with quadratic performance indexes is described in this paper. The phase-variable canonical form is used to develop a new type of optimality criterion for equivalence. This concept leads to a substantial simplification of the Riccati equation. The simplified matrix Riccati equation is of the form  $\dot{P} = Q - P A^T - A P + P P$  for any problem of a given order, say,  $n$ , and contains only  $n$  nonzero forcing functions. That is, it always corresponds to a set of constant-coefficient scalar differential equations; in various  $n$ th-order problems the  $n$  nonzero forcing functions and the terminal conditions simply assume different forms. In a very strong sense, this simplified matrix Riccati equation is the most general possible Riccati equation arising from optimization problems. The method is developed for general time-varying systems with finite terminal time. It is developed also for the important special case of time-invariant systems with infinite terminal time.

**Support:** This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 and in part by the Advanced Research Projects Agency of the Department of Defense under the same contract monitored by the Office of Naval Research.

**Remarks:** This paper was presented, at the Winter Annual Meeting of the American Society of Mechanical Engineers, New York, New York, December 1-5, 1968. The paper was published in the Journal of Basic Engineering, Transactions of the ASME, June 1969, pp. 149-154.

**Technical Report:** CHEMICAL RANGING AND TRACKING

**Author:** Dr. R. C. Gesteland

**Date:** July 31, 1969

**Abstract:** The vertebrate nose is unrivaled in its sensitivity to a wide range of trace chemical substances occurring in air and water. At present there are no useful, broad spectrum instruments suitable for real time-chemical detection and analysis. Significant advances in understanding the chemical basis of the olfactory receptor membrane, and in relating the signalling parameters of nervous activity to receptor stimulation have been made during the period of this grant. In addition, a comparative histological study has turned up a vertebrate nose with very large receptor cells, which should allow us to monitor cell function with intracellular microelectrodes for the first time. Finally, several methods have been developed for using an electrochemical cell as an indicator of the presence of trace contaminants in a flowing stream. Each of these projects is discussed at some length in the following pages.

**Support:** This work was supported in part by the U. S. Army Research Office, Durham, North Carolina under Grant Number DA-ARO-D-31-124-G991, in part by the U. S. Air Force under Contract Number F33615-67-C1497; in part by the National Institute of Health under Grant Number 1-R01-NB-06063, in part by the National Science Foundation Predoctoral Fellowship Program, in part by the National Institute of Health Training Grant and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0020-A03.

Technical Paper: ON THE DISTRIBUTIONAL FORMULATION OF MAXWELL'S EQUATIONS

Authors: Dr. O. Ben-Dov and Dr. R. E. Beam

Abstract: The ambiguity between a distributional formulation of Maxwell's equations and the Maxwell's equations themselves is discussed. A way to overcome this ambiguity is shown by defining a vector distribution with the aid of a test dyadic.

Support: This work was supported in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003.

Remarks: This paper was published in the IEEE Transactions on Antennas and Propagation, Vol. AP-16, No. 4, July 1968, pp 500-501.

Technical Paper: STUDIES OF SINGULAR SOLUTIONS IN DYNAMIC OPTIMIZATION  
--Optimal Singular Design of a Plug Flow Tubular Reactor

Authors: Dr. D. Y. C. Ko and Dr. W. F. Stevens

Abstract: This paper presents the details of an application of the method of solution presented in a previous companion paper to the optimal design of a tubular reactor. It is shown that, in general, if the reactor is "sufficiently long", the optimal heat transfer coefficient distribution always ends with a singular control section. Before singular control is applied, the control may or may not switch, depending on the initial conditions and the constraint on the control variable. Numerical results between the new method and a gradient approach.

Support: This work was supported in part by the Union Carbide Corporation, in part by the Corn Products Company and in part by the Joint Services Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003.

Remarks: This paper was presented as part of the Symposium on Optimization of Reaction Systems at the Cleveland Meeting of the American Institute of Chemical Engineers, May 4-7, 1969. It was published in the AICHE Journal, Vol. 17, No. 1, January 1971, pp. 160-166.

Technical Paper: STATISTICAL PROPERTIES OF MULTI-CATEGORY PATTERN CLASSIFICATION BASED ON MEAN-SQUARE ERROR CRITERION

Authors: Dr. S. S. You and Dr. P. C. Chuang

NOT REPRODUCIBLE

Abstract: It is shown that the dispersion matrix, which is used to find the optimal weight matrix for the multi-category pattern classifier, is the sum of the average covariance matrices of the pattern classes and the generalized covariance matrix defined for the set of means of the pattern classes.

Support: This paper was supported in part by the U. S. Air Force Office of Scientific Research under Grant AF-AFOSR-1292-67 and in part by the Joint Service Electronics Program (U. S. Army, U. S. Navy and U. S. Air Force) under Office of Naval Research Contract Number N00014-66-C0030-A03.

Remarks: This paper was published in the IEEE Transactions on Information Theory, Vol. IT 14, Number 5, September 1968, pp. 778-780.

Technical Paper: ELECTRON BEAM SENSING OF SURFACE ELASTIC WAVES.

Authors: Dr. M. Epstein, Dr. S. G. Joshi, R. J. Serafin and Dr. A. P. Van den Heuvel. Dr. Epstein is at Northwestern University. Dr. S. G. Joshi, R. J. Serafin and Dr. A. P. Van den Heuvel are at Illinois Institute of Technology.

Abstract: A method of readout of elastic surface waves by means of an electron beam is described. The signal detected by the beam is obtained directly in the form of the envelope of an RF pulse at a carrier frequency of six MHz.

Support: This work was supported in part by the Advanced Research Projects Agency of the Department of Defense, and was monitored by the Office of Naval Research under Contract Number N00014-67-A-0356.

Remarks: This paper was published in the Proceedings of the IEEE, Vol. 58, No. 2, February 1970, pp. 253-4.

Technical Paper: A PULSED GaAs LASER SYSTEM FOR SEMICONDUCTOR CARRIER LIFETIME MEASUREMENTS

Authors: Dr. C. R. Kannewurf and M. E. Motamedi

Abstract: An electronic system is described for controlling a semiconductor laser, which is convenient for measuring semiconductor carrier lifetimes of 500 ns or greater by the photoconductive decay method with the laser providing the sample illumination. Particular attention is given to the circuit of the power pulse generator, which was designed to operate the laser above threshold and to provide a means of controlling the output intensity of the laser radiation.

Support: This work was supported in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy and U.S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003.

Remarks: This paper was published in the IEEE Transactions on Instrumentation and Measurement, Vol. IM-19, Number 1, February 1970, pp. 6-9.

Technical Paper: ON THE CHARACTERISTICS OF LINEAR OPTIMAL SYSTEMS

Author: Dr. W. J. Rugh

Abstract: The characteristics of linear systems with constant inputs which are optimal with respect to an infinite terminal time, quadratic performance index are studied. An especially simple standard form of the algebraic Riccati equation is used to determine relationships between the performance index parameters and the normalized characteristic polynomial of the optimal system. The parameters corresponding to several standard forms of this polynomial are ascertained. Design guidelines are then given which allow the correlation of performance index parameters with the shape of the step function response.

Support: This work was supported in part by the U.S. Army, Navy, and Air Force in a Joint Service Electronics Program under Office of Naval Research Contract No. N00014-67-A-0356-0003, and in part by the Advanced Research Projects Agency of the Department of Defense under the same contract, monitored by the Office of Naval Research.

Remarks: This paper has been accepted for publication in the Journal of the Franklin Institute.

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Technical Paper: WIDE-BAND TRANSDUCER FOR SURFACE ELASTIC WAVES

Authors: Dr. M. E. Motamedi and Dr. M. Epstein

Abstract: A log-periodic interdigital transducer for surface elastic waves was fabricated and shown to have good agreement with an approximate analytical model. Experiments were performed on a mechanically variable delay line.

Support: This work was supported in part by the National Science Foundation under Grant GK-10808 and in part by the Advanced Research Projects Agency under ARPA Order Number 1129 and was monitored by the Office of Naval Research under contract number N00014-67-A-0356.

Remarks: This paper was published in Proceedings of the IEEE, Vol. 58, No. 11, November 1970, pp. 1870-1.

Technical Report: 70-101

Title: SURFACE MICROACOUSTICS

Author: Dr. M. Epstein

Date: September 1970

Abstract: This report is an introduction to the field of surface microacoustics. It includes a review of the present state-of-the-art and an annotated bibliography.

Support: This work was supported by the Advanced Research Projects Agency under ARPA Order Number 1129 and was monitored by the Office of Naval Research under contract number N00014-67-A-0356.

Technical Paper: A METHOD FOR THE DESIGN OF STABLE NONLINEAR FEEDBACK SYSTEMS

Author: Dr. G. J. Murphy

**Abstract:** One of the contributions of this paper is the derivation of a new modification of Popov's criterion that eliminates the need to consider the different cases separately. That is, in using the new form of stability criterion, the designer need not be concerned with the question of whether the system is a direct control system or an indirect control system or the question of whether it represents the principal case or one of the particular cases.

A second advantage of the new form of stability criterion is that it results in a smaller number of distinct conditions that must be checked than does the most attractive of the methods previously described. The test for stability is thus reduced to the simple procedure of determining that the open-loop system is controllable, observable, and stable, and then mapping a specified curve from the s-plane to a G-plane, just as one does in investigating the stability of a linear feedback system by Nyquist's criterion.

Another contribution of this paper is the generalization of the earlier frequency-domain tests for stability to encompass a previously published extension of Popov's criterion, which was too cumbersome to be of practical use before this new procedure in the frequency domain was conceived. It is shown, also, that this far more powerful criterion can be employed relatively easily in conjunction with the new design procedure presented in this paper.

The new procedure is simplified even further for use in those cases in which the simple Popov criterion is adequate; and a new chart and a new test chart are to be used by a designer in choosing a linear compensator, given the nonlinear element and the linear plant, or in choosing a nonlinear element, given the linear plant (and, possibly, a linear compensator) are described.

The paper includes also an example illustrating the use of the new form of stability criterion and the new design procedure to design a stable nonlinear feedback system.

**Support:** This work was supported in part by the Air Force Office of Scientific Research under Grant AF-AFOSR 270-65 and in part by the Joint Services Electronics Program (U.S. Army, U.S. Navy and U.S. Air Force) under Office of Naval Research Contract Number N00014-67-A-0356-0003 Modification AB.

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## 13. ABSTRACT

This report is a summary of the research conducted in Northwestern University's Information-Processing And Control Systems Laboratory from May 1, 1966 through April 30, 1971 under Contract Numbers N00014-66-C0020-A03 and N00014-67-A-0356-0003.

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